# A Signal Transmission Technique for Stability Analysis of Multivariable Non-linear Control Systems

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## 100 Word Abstract:

Among the difficulties associated with multivariabe, non-linear control systems is the problem of assessing closed-loop stability. Of particular interest is the class of non-linear systems controlled with on/off actuators, such as spacecraft thrusters or electrical relays. With such systems, standard describing function techniques are typically too conservative, and time-domain simulation analysis is prohibitively extensive. This paper presents an open-loop analysis technique for this class of non-linear systems. The technique is centered around an innovative use of multivariable signal transmission theory to quantify the plant response to worst case control commands. The technique has been applied to assess stability of thruster controlled flexible space structures. Examples are provided for Space Shuttle attitude control with attached flexible payloads.

## Extended Abstract:

### Problem Definition

Among the difficulties associated with multivariabe, non-linear control systems is the problem of assessing closed-loop stability. Of particular interest is the class of non-linear systems controlled with on/off actuators, such as spacecraft thrusters or electrical relays. In this class of systems, control is typically achieved through a series of pulsed commands to the actuators. This invalidates standard linear frequency domain analysis which assumes control actuations are continuous and sinusoidal. Pulsed actuator commands are typically generated through the use of switching deadbands to avoid actuator chatter. Two types of control techniques are common with these systems, pulse modulation, and logical controllers. Pulse modulation approximates a linear system by controlling pulse widths or frequencies in proportion to feedback errors. The deadbands are then employed on the duration or frequency of the pulse commands to avoid chattering. Logical controllers, on the other hand, typically determine commands based on the relationship of control errors to logical switch curves, and the deadbands are directly implemented in the switching logic. The Space Shuttle control law shown in Figure 1 is a complex implementation of a logical control law.

Although these control techniques have been successfully applied to many spacecraft systems, stability analysis of the closed loop system has remained a significant design challenge. Standard nonlinear analysis techniques, e.g., describing function analysis, have been shown to be conservative requiring either highly conservative design margins (and/or mission constraints) or significant time-domain simulation analysis to verify stability and performance.

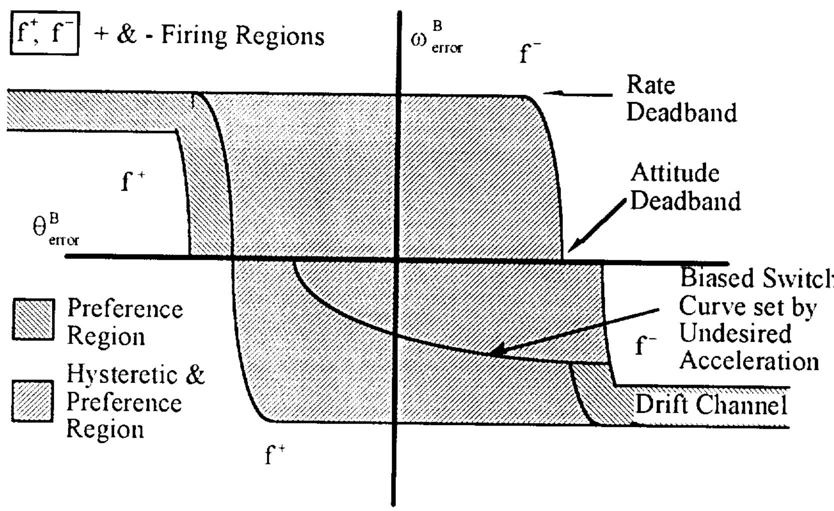



Figure 1. Space Shuttle Control System with Logical Control Law.

The stability analysis problem is further complicated when the controlled channels of the linear plant are coupled in a multivariable system. Many techniques and tools have been developed for the analysis of linear systems under these conditions which rely on Singular Value Decompositions (SVDs) of the plant transfer function matrix. Unfortunately, these and the aforementioned non-linear analysis techniques have not seen practical use for this class of non-linear systems. One key reason is that singular value magnitudes represent the steady-state plant response to a sinusoidal input at a particular frequency. For on-off actuators, however, the plant is driven by pulses that tend to spread the power of the response over multiple frequencies. This reduces the gain at the modal frequency as seen in Figure 2 which shows the frequency response to various types of inputs. The over-prediction of the response from the singular value gain results in overly pessimistic stability predictions.

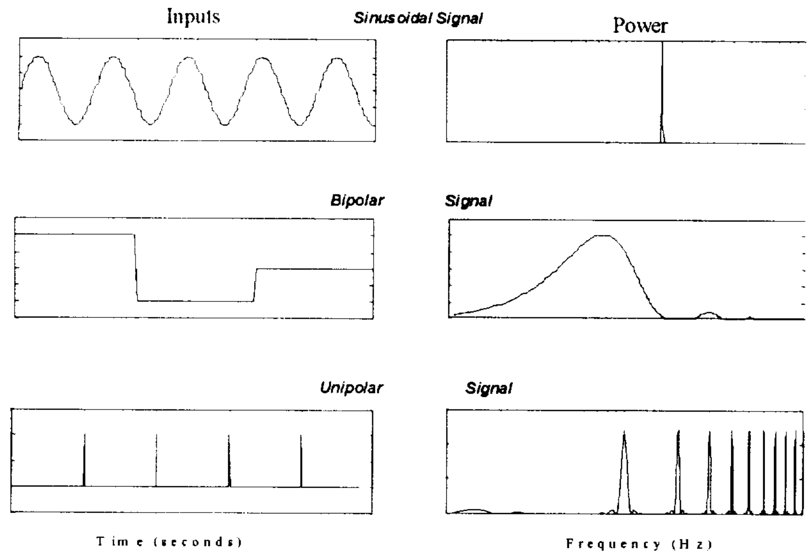



Figure 2. Effects of Pulsed Inputs on System Response

A final motivation for the new technique is that, particularly with spacecraft, plant characteristics may vary significantly, and the plant is often quite uncertain. This means that stability analysis must be repeated for each new plant or payload configuration, and each analysis must account for uncertainties. For these reasons, stability assessment by time-domain simulation becomes extremely difficult.

## **Problem Solution**

This paper develops an open-loop analysis technique to assess control system stability for this class of non-linear systems. It derives the worst case plant dynamic response (i.e., input to the controller) to a defined set of control commands (i.e., the output of the controller). These worst case inputs can then be compared to the deadbands to determine if sufficient excitation could exist to induce unstable behavior. The technique relies on the ability to characterize the expected performance of the system under stable conditions. It then applies multivariable signal transmission theory to quantify the worst case response of the system under these conditions for comparison to the deadbands.

To account for the on/off nature of the control system commands, multivariable signal transmission theory is applied to determine a nonconservative bounded response to a given set of input driving functions. Multivariable signal transmission is given as

$$\Phi_{yy}(\omega) = G(j\omega)\Phi_{uu}G^{*}(j\omega), \tag{1}$$

where $G(jw)$ is the plant and $\Phi_{uu}$ and $\Phi_{yy}$ are the input and output power spectral densities (PSD) respectively. The output signal variance is defined as

$$\sigma_{yy}^{2} = \frac{1}{\pi}\int_{0}^{\infty} Trace\left[\Phi_{yy}(\omega)\right]d\omega, \tag{2}$$

which can be derived as a function of the plant and input PSD as

$$\sigma_{yy}^2 = \frac{1}{\pi}\int_0^{\infty}\left\{\sum_i \sigma_i^2\left[\sqrt{\Phi_{uu}(\omega)}G^*(j\omega)\right]\right\}d\omega, \tag{3}$$

where $\sigma_i$ is the $i$th singular value and $\sigma_{yy}^2$ is the variance of the output signal y(t). To allow comparison of the signal amplitude to the control deadbands, the output signal variance must be converted to a sinusoidal equivalent amplitude corresponding to the given input signal by

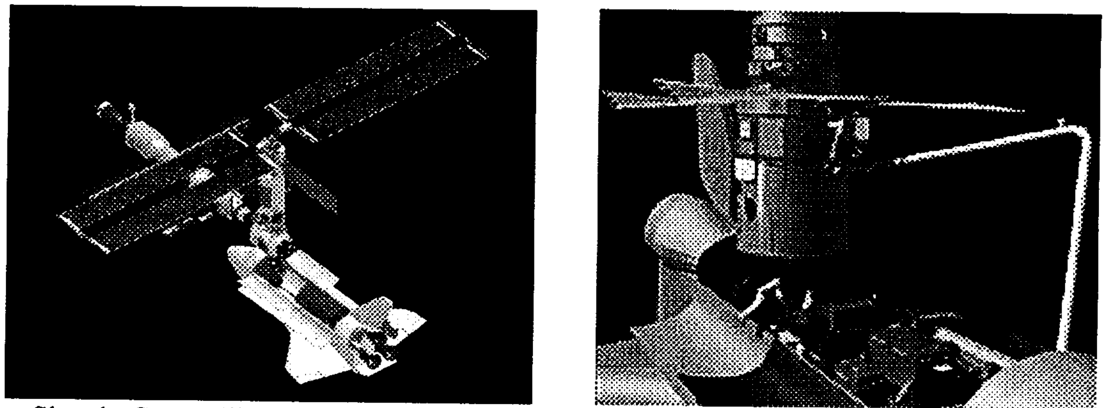
$$A\big|_{\Phi_{uu}} = \sqrt{\sum_i A_i} = \sqrt{2}\sigma_{yy}, \tag{4}$$

where $A$ represents the 2-norm of the output signal amplitude vector.

The bounded system response is obtained by sweeping over all a space of command directions for each frequency of interest. The result is a set of worst case amplitudes at each of the modal frequencies of the plant. Once this frequency response has been derived from signal transmission, stability is assessed by comparing the worst case response to switch curve deadbands. If the resultant flexural rate from the bounded worst case pulse pattern can not exceed the rate deadband, an unstable limit cycle cannot be induced.

## Application

The new technique has been applied to the certification of the Space Shuttle on-orbit control system during deployed payload operations. Quite often during these operations large flexible structures are either directly mated to the Shuttle (Mir, International Space Station) or deployed on the remote manipulator (Hubble Space Telescope, ISS components) as in Figure 3. For each of these configurations the control system must be certified to remain stable for planned and contingency operations, under varying degrees of uncertainty and multiple thruster control modes. The simulation analyses of these configurations would be prohibitive. Additionally, notch filters were employed into the Shuttle autopilot to provide a means for stabilization. The simulation techniques do not adequately provide quantitative requirements for the design of these filters. The new techniques have been shown to provide adequate stability margins and performance. This section provides a summary of the application of the proposed techniques to the analysis of Shuttle control system stability and design of notch filters to stabilize the control system. A simplified overview of the control system is presented and two flight configurations are analyzed. Finally, flight results from the assessed mission are provided to demonstrate the suitability of the proposed technique.



Shuttle Controlling Attached Station. Shuttle Controlling Hubble on Manipulator.

Figure 3. Shuttle RCS Control of Varying Payloads